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A Volatility Based Modified Black Scholes Approach to Price Options

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Key words: Black Scholes equations, computational finance, implied volatility, stochastic asset model

Author Summary: The purpose of the project was to help reduce the uncertainty in the stock market, in order to prevent future market crashes. A set of equations, the Black Scholes model, is able to price stock assets in the future, providing useful information about market conditions. In order to improve the accuracy of the readings of the Black Scholes, random data sets were inputted, and the results were analyzed. In the end, volatility, a variable which quantifies how unstable the market is, was found to be the most important factor in the Black Scholes model.

Abstract

The goal of the project was to improve the Black Scholes model accuracy in pricing options at higher values of volatility. The market is constantly overwhelmed with uncertainty, within lies the potential of an asset's price to rise or fall significantly. This brings about the concept that the market moves stochastically that is randomly. This variability of the market is quantified as the term, volatility. Using the volatility values, stochastic asset models can be formed, which are capable of finding the values of assets in the future. One of the most prominent stochastic asset models is the Black Scholes option pricing model. Formulated by Fischer Black and Merton Scholes, this option pricing model is the most widely utilized model in the market. Through a series of random value testing, the volatility was found to be the most significant factor in the Black Scholes model. In conclusion, a basic computer simulation model was developed, using the Black Scholes model to price options.

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Introduction

The impetus for the project was to develop a user-friendly tool to enable simpler investing for average people with no financial knowledge to be able to incorporate the higher level Black Scholes model into making a tactical fiscal decision. This would be achieved through improving the accuracy rate of the Black-Scholes model to enhance decision-making based off of the stock market.

Within the stock market, there are two types of stocks. The first type is defensive; the other is cyclical. Defensive stocks are utilities and food. Utilities and food are defensive because the demand does not change much. As a whole, defensive stocks' prices are resilient regardless of economic crisis or boom. Cyclical stocks are basic materials, capital goods, communication, consumer cyclical, energy, financial, healthcare, technology and transportation. These are cyclical stocks since the demand can vary often and they may easily decrease during economic boom. These stocks are also affected more by the state of the economy [1]. A higher level of finance will lead to the option and derivative market, both of which are types of contracts which allow for the buyer to exercise the right to buy or sell an asset at a specified strike price, during the period of time before expiry. The strike price is a designated price that is agreed upon by the buyer of the option and the asset holder/buyer. The buyer of this contract is required to pay a fee called a premium [2]. There are two types of options: call options and put options. A call option contract is where the purchaser of the call option is allowed to buy the certain asset from the seller of the call option, at the strike price, but the call option holder may opt not to do so [3]. On the other hand, the put option contract is where the purchaser of the put option is allowed to sell a certain asset from the writer of the put option, at the strike price, but is not obligated to do so [4].

Within the options market, there are two distinct types of options, America and European-style. European style options function in that the right to buy or sell the specific asset only applies for the predetermined expiration date of the option, whereas, American-style options allow for the purchaser to exercise the right anytime during the timeframe of the purchase and the expiration date [5].

The Black Scholes option pricing model, utilizes the factor of the stock price, strike price, risk-free interest rate, volatility, and time until expiry. The following shows the equation

$$d_1 = \frac{\ln\left(\frac{S}{k}\right) + \left(r + \frac{s^2}{2}\right)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

Where S is the current stock price, k is the strike price, r is the risk-free interest rate, t is the time until expiry, and s is the standard deviation (volatility) of stock returns.

The Black Scholes equations also account for six limitations as follows [6]:

1) The stock pays no dividends
2) European exercise terms are used
3) Markets are efficient
4) No commissions are charged
5) Interests rates are constant
6) Returns are lognormally distributed

The values d_1 and d_2 are then entered into the standard normal distribution function, shown below:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\frac{x^2}{2}}dx$$

$$\mathcal{N}(x) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{x^2}{2}}dx$$

The values d_1 and d_2 are entered into this format to derive the value of the call option:

$$\mathcal{N}(d_1)S - \mathcal{N}(d_2)ke^{-rt}.$$

Within the Black Scholes Option Pricing Model, the current stock price is defined as the price of which the stock presently is at. The strike price is set price in which the option matures and offers the contract holder to exercise the right to buy/sell a certain commodity at that price. The risk-free interest rate is commonly notated at the federal Certificate of Deposit (CD) interest rate, in which assuming that investing money in a CD account holds no risk. The time until expiration is the deadline that upon maturity, neither buyer nor seller of the financial derivative is obligated to buy or sell the asset. The volatility/standard deviation is the term used to calculate the possibility of a stock changing price [7].

Additionally, the Black Scholes Model utilizes volatility which has several different variations: actual historical volatility, actual future volatility, historical implied volatility, current implied volatility, and future implied volatility. Actual historical volatility involves the volatility of some financial instrument over a set period of time within the past. Actual future volatility is related to the volatility calculated over the current time to a set date in the future, commonly at the expiration of a financial derivative. Historical implied volatility is the volatility calculated from previous data, of some asset within an option contract, inputted into an option pricing mode, while the current implied volatility is where the volatility is presently standing at. The future implied volatility is a similar type of volatility, but utilizes prediction tools to calculate of that specific option [8].

The Black Scholes implements a specific type of volatility, called the Black-Scholes volatility which utilizes a lognormal calculation for annualized volatility, and has several major portions: Delta(Δ), Gamma(Γ), Theta(θ), Vega($^{\delta V}/_{\delta\sigma}$), and Rho($\rho$). Delta is the ratio of the change in an option's price to a given change in the price of the underlying asset or instrument. Gamma is the rate of change of an option's delta with respect to a change in the price of the underlying assets or instrument. Theta is the rate of change of an option's price with respect to time the closer the option is to expiration, the greater the value of theta. Vega is the rate of change of an option's price with respect to the change in the volatility of the underlying asset or future. Rho is the rate of change of an option's price with respect to the change in the risk-free interest rate [9].

A beta coefficient is a number that is based off of the co variance and variance of the rate of returns on a stock, and can be utilized to compare the volatility of an underlying asset to a major index and determine the classification of an object's volatility [10].

Beta Value	Meaning
$\beta < 0$	Opposite direction relative to index
$\beta = 0$	No correlation to index
$0 < \beta < 1$	Less movement, same direction as index
$\beta = 1$	Same movement, same direction as index
$\beta > 1$	More movement, same direction as index

Derivative and integral calculus play an important part in analyzing functions and equations, where finding derivatives can help find critical points, such as turning points and local extrema. Finding integrals can assist in finding the area beneath a curve and finding the equation obtained before taking the derivative. Derivatives can be expressed through the basic limit definition as:

$$\frac{d}{dx}f(x) = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

this representing, the slope of a tangent line to some function. While the definite integral can be expressed as:

$$\int_a^b f(x)dx$$

this representing the area beneath the curve over the interval [a,b] for f(x). Additionally, the Fundamental Theorem of Calculus states that such an expression can be written as F(b)-F(a), where F(x)=$\int f(x)dx$, in addition it mentions that $\frac{d}{dx}\int f(x)dx = f(x)$, as the two operations derivation and integration are inverses [11].

The hypothesis was if the Black Scholes equations used a MacLaurin series within the normal distribution function, then it would be possible to calculate a volatility value, which would have a higher accuracy rate than the previous "Black Scholes volatility" and the implied historical volatility by one standard deviation because introducing the MacLaurin series allows for easier fine-tuning of the individual components in the Black Scholes equations.

Materials

The materials used within the experiment were a HP EliteBook, 802.11n wLAN network adapter, Microsoft Office 2010, Microsoft Visual Studios 2012, Google Chrome, and Windows 7 Ultimate.

Methods

The Black Scholes Option Pricing Model was converted into VB.net, and a Microsoft Visual Studios 2012 Windows Form Application was developed, entitled Black Scholes Simulator. The first form contained a set of labels and corresponding textboxes, with the variables: stock price, strike price, risk-free interest rate, volatility, and time until expiry, and the variables were globally declared in Form 1. Two buttons, named 'confirm' and 'cancel', were inserted with the respective options to confirm the variables' values and to close the form. The confirm loop was linked to an external form, Form 2, which was the output form, with the single button to calculate the Black Scholes Value according to the options pricing model, and would display the value in a label.

Subsequently, the options pricing model was analyzed through a series of substitutions, to determine which variables to alter to properly manipulate the model. A series of five numbers were collected from a random number generator, they were: 48, 54, 6, 8 and 52. The values of these were altered by ±50% to determine the effect of each variable. The trials were ran by controlling all the variables except the one being modified for example the data set would become: 24, 54, 6, 8, and 52, were the to be altered set of option data random integers. Following the Black-Scholes analysis, the volatility was examined to determine the correlation to the variables and the original model.

The dependent variables in this experiment are the price of the call or put option. The control variable is the Black Scholes model being utilized. The independent variables are volatility, stock price, strike price, time until expiration, and the risk-free interest rate.

Results

Table 1 shows the various sets used the computer analysis of the Black-Scholes Model, where the original Set 1, had the respective variable values increased or decreased by 50% and inputted into the computer simulation. Table 2 displays the results of the data input from Table 1, with the sets matching to its call or put option value. Table 3 analyzes the correlation of each variable to the call and put option values, describing the qualitative data of how the value of the predicted call or put option had changed. Table 4 depicts the overall change that a 50% increase or decrease in the variable had on the option price. The headings call: +50% translated to the price of the call option after the specified variable on the left had been increased by 50%, and the same meaning follows. The heading Avg change symbolized the average percentage calculated per row, using the absolute value for each percentage as change in either direction is the same numerical value.

Graph 1 displays the Gaussian error function, which is a vital function in calculating the Black-Scholes value. The trending of the graph moves upwards approaching 1 for all positive values of x, hitting a horizontal asymptote at y=1, as x approaches infinity. The inverse follows for the negative values of x, where it approaches −1, hitting the horizontal asymptote of y=−1 as x approaches negative infinity. A similar pattern is apparent in the normal distribution function, however, it is shown that the horizontal asymptotes lie at around y=±0.35.

Figure 1 shows the integral calculation results for the Gaussian error function, erf(x). Where, the integral was shown to be the reciprocal of the erf function, additionally, the second half of Figure 1 the direct integration by rules is done, where the function is the same, with an added indefinite constant. Figure 2 shows all the volatility formulas used in the Black-Scholes modification process. The original model utilized the natural logarithm formula to calculate the volatility and utilizes the standard deviation formula, multiplying it by the square root of 252 as that is the rough estimate of the number of trading days in a year, as it calculates the annual volatility. The variables p_c and p_y stand for the price of the stock on the current date and the previous trading day, for example the formula would calculate the volatility of the three prices (1,2,3) using the data set of $(\ln{}^{3}/_{2}, \ln({}^{2}/_{1}))$.

Table 1. This table depicts the original randomized numbers in Set 1, and the remaining sets show the manipulated sets for analysis.

	Current Price	Strike Price	Time to Expiry	Interest Rate	Volatility
Set 1	48	54	6	8	52
Set 2	24	54	6	8	52
Set 3	72	54	6	8	52
Set 4	48	27	6	8	52
Set 5	48	81	6	8	52
Set 6	48	54	3	8	52
Set 7	48	54	9	8	52
Set 8	48	54	6	4	52
Set 9	48	54	6	12	52
Set 10	48	54	6	8	26
Set 11	48	54	6	8	78

Table 2. This table matches the Table 1 data sets and shows the value of the call and put options from the Black Scholes Simulator.

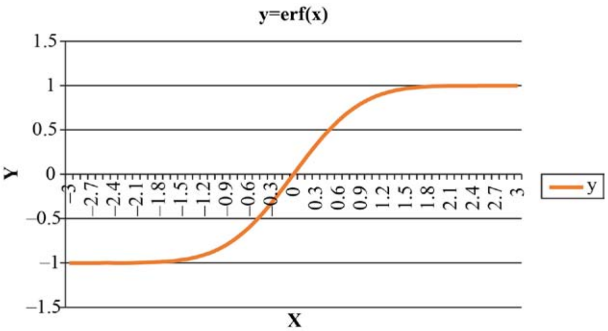
	Call	Put
Set 1	27.33	12.744
Set 2	9.3466	18.761
Set 3	48.003	9.4176
Set 4	34.949	3.6564
Set 5	22.294	24.415
Set 6	18.618	13.096
Set 7	33.038	11.323
Set 8	24.371	18.849
Set 9	30.159	8.4436
Set 10	18.991	4.4053
Set 11	34.531	19.945

Table 3. This table translates the data in Table 2 into trendings.

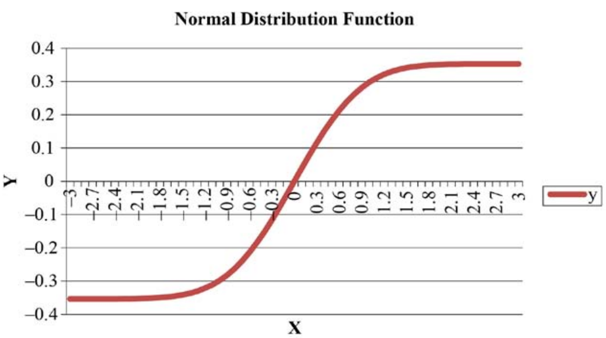
	Type of Change	Call Change	Put Change
Set 1	n/a	n/a	n/a
Set 2	Decrease	Decrease	Increase
Set 3	Increase	Increase	Decrease
Set 4	Decrease	Increase	Decrease
Set 5	Increase	Decrease	Increase
Set 6	Decrease	Decrease	Increase
Set 7	Increase	Increase	Decrease
Set 8	Decrease	Decrease	Increase
Set 9	Increase	Increase	Decrease
Set 10	Decrease	Decrease	Decrease
Set 11	Increase	Increase	Increase

Table 4. This table analyzes the correlation between the variable and the Black Scholes model.

	Call: +50%	Call: –50%	Put: +50%	Put: –50%	Avg change
Stock Price Change	75.64%	–65.80%	–26.10%	47.21%	53.69%
Strike Price Change	–18.40%	27.87%	91.58%	–71.30%	52.29%
Time to Expiry Change	20.88%	–31.88%	–11.15%	2.76%	16.67%
Interest Rate Change	10.35%	–10.82%	–33.74%	47.90%	25.70%
Volatility Change	26.34%	–30.51%	56.50%	–65.40%	44.69%



Graph 1. This line graph displays the Gaussian error function which is utilized in the normal distribution function



Graph 2. This line graph displays the normal distribution function which is utilized in the Black-Scholes Model.

$$\int_0^x e^{-x^2} dx = \frac{\sqrt{\pi} * \mathrm{erf}(x)}{2}$$

$$\int_0^x e^{-x^2} dx = e^{-x^2} + C$$

Figure 1. This figure displays the calculations to find the integral portion of the Gaussian error function.

The next equation was of the beta coefficient, where it shows the trending of an asset relative to an index, where it takes the covariance of the r_a and r_b, where they are the rate of return on an asset and the rate of return on a portfolio benchmark, respectively.

$$\textit{Black Scholes volatility}: \sqrt{252 * \frac{\sum_c^y \ln(\frac{p_c}{p_y})}{C-Y}}$$

$$\textit{Beta Coeffcient}: \frac{cov(r_a r_b)}{var(r_b)}$$

$$\textit{Current Implied Volatility}: x_{i+1} = \frac{y_i - p}{v_i}, \textit{until} \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \leq \varepsilon,$$

$$\textit{where } x_i = IV$$

$$\textit{Future Implied volatility}: \frac{IV_b T_b - IV_1 T_1}{T_b - T_1}$$

Figure 2. These set of equations depict the various volatility formulas utilized.

The next formula calculates the current historical implied volatility, where p, x_i, y_i, and v_i represent the price of the asset, volatility, theoretical value of the option at the volatility of x_i, and degree of accuracy. The future implied volatility formula calculates the volatility of the next "period" of time, where the previous historical data is of the same length. The variables, IV_b, IV_1, T_b, and T_1 within the formula stand for the implied volatility of both time periods, implied volatility of the previous time period, duration of time for both time periods, and the duration of time for the first time period.

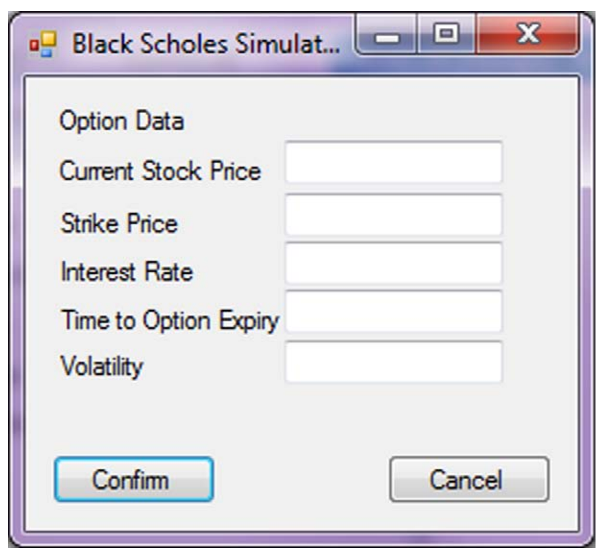


Figure 3. The main menu form of the Black Scholes simulator.

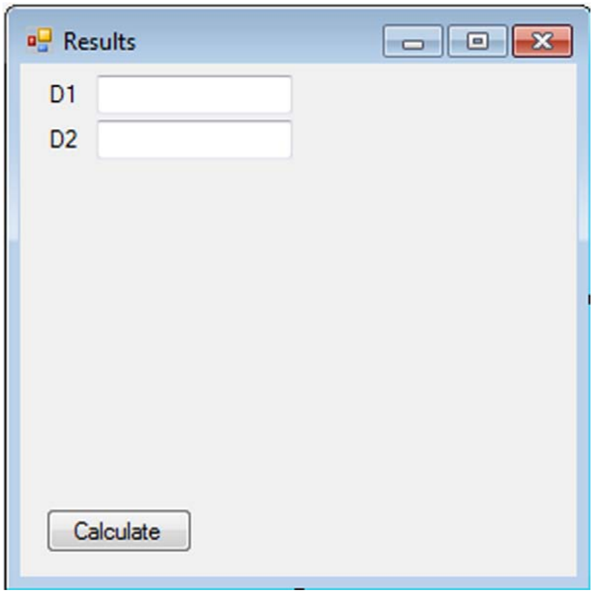


Figure 4. The output form of the Black Scholes simulator.

Figures 3 and 4 are screenshots of the computer simulation form, where there are buttons to allow the user to confirm the data and results, and textboxes to input numerical values for the Black Scholes model.

Discussion

The conclusion was that the hypothesis was inconclusive because the programming within Visual Basic 2012 did not allow for the integration of the Gaussian error function, thus severely limiting the functionality and capability of the computer simulation. However, the hypothesis was shown to be inconclusive rather than not accepted, as the clear relation between volatility within the Black Scholes model was demonstrated.

This clear relationship is shown in Table 4, as the volatility is the most influential variable that cannot be set by the user. This is so, due the fact that stock price, strike price, and time to expiration are all selected by the user, while risk-free interest rate and volatility is determined by the market and economy; however, the risk-free interest rate generally shows little movement and was demonstrated to have the second least effect on the Black-Scholes price. Adding onto this, is the major error in utilizing the Black-Scholes volatility, as it is annualized from historical data, and volatility is very unpredictable in its movement, and is prone to changes despite the historical references, however, using the forecasting volatility using the implied volatility ratio and the Vega function as a derivative with respect to volatility, forecasting volatility is shown to be much more accurate and thus more likely to correctly model the American-style options.

Next off, there is the other noticeable issue, where the indefinite constant, found in Figure 1 was indeterminate. Based off of the Fundamental Theorem of Calculus the value of $\int_0^x e^{-\frac{x^2}{2}}dx$ should have been equal to $e^{-\frac{x^2}{2}} - 1$, however, the results did not match up, so the integral calculated was invalidated.

The limitations upon the experiment include the time, programming language/ environment, and financial resources. As, with more time allowed, the program could have been more enhanced and more precise, deviating much less from the actual option price. Given additional time, the program could have incorporated more details and overcome the poor programming language choice. Because of the usage of VB.net, there was no clear way to calculate the normal distribution, as several failed attempts included looping the Gaussian error function, or embedding an excel spreadsheet. Lastly, the financial resources were severely limiting as with more capital, better option data could have been procured as well as an advanced, expanded programming library for more utility and functionality in the computer simulation.

Future improvements include utilizing external variables to more accurately model the real world scenarios and thus improve the volatility calculations. External variables such as ongoing crises, unexpected plummets or rises in the stock market, could all attribute the sudden bursts of change in volatility, and overall better prediction models of these factors would increase accuracy and precision of the modified Black Scholes option pricing model. Future applications also could expand to a more fine-tuned stock price trending model, and economic regression model, as volatility is one of the main defining factors in determining the outcomes.

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