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Chronicle of The New Researcher

The Generation and Analysis of Waves with Varying Nonlinearity through a One-Dimensional Chain of Spheres

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Key words: nonlinear wave, soliton, linearity, material acoustics

Author's Summary: The research was conducted to determine how to produce highly nonlinear waves. Nonlinear waves, solitons specifically, can travel extremely long distances and hold high amounts of energy. Steel spheres were set end-to-end and struck to create a wave that would travel from one sphere to the next. A crystal sensor was epoxied between two sphere-halves to determine the traveling wave's linearity. The data revealed that higher striking forces and higher compression on the spheres produced more nonlinear waves. The results could improve medical devices utilizing solitons in place of ultrasound.

Abstract

Many waves formed in nature are primarily linear and have a property called dispersion causing different waveforms to separate and lose amplitude. Solitons can retain their pulse form as they propagate and cancel out that dispersion, making nonlinear waves useful for transferring energy. Waves traveling through a chain of spheres would display nonlinear behaviors when struck with enough force causing separation in the chain; the sound would not reflect back through the system disrupting the wave. Also in a chain of spheres, the waves traveling within would display nonlinear behaviors; the stress from the spheres' masses would cause deformation in the chain and waves traveling through them become nonlinear. The first trials tested wave behaviors with different amounts of striking force. A test sphere was placed atop the chain and struck with various forces. The second trials tested wave behaviors with a consistent force, changing the location of a test sphere in the chain. The third trials tested wave behaviors with a consistent force, changing the location of a test sphere in the chain and applying varying forces to the chain. The fourth trials repeated the methodology of the three previous trials, but extended the chain threefold. This research was considered successful since both hypotheses were accepted and supported by the nonlinearity of higher and lower forces and the nonlinearity of top and bottom test sphere positions being statistically different. Focused solitons provide many innovative practices such as replacing dangerous radiation therapy treatments or High-Intensity Focused Ultrasound, providing various military applications, and creating new communication avenues.

Citation: Thorsen M. Wehr, (2014) "The Generation and Analysis of Waves with Varying Nonlinearity through a One-Dimensional Chain of Spheres" *Chronicle of The New Researcher* 1(1): 27-34, doi:10.1511/CTNR.2014.1.27

Editor: Richard Wiggins PhD, Sigma Xi, Research Triangle Park, NC USA

Received August 15, 2014; **Accepted** October 21, 2014

Funding: The authors have no funding or support to report.

Competing Interests: The authors have declared that no competing interests exist.

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Introduction

Many waves formed in nature are primarily linear following linear rules of superposition and scaling. Natural media through which waves travel typically have a property called dispersion where the sinusoids of different frequencies that sum together to form the waveform travel at different speeds [1]. This causes the waveforms to separate, or disperse (Figure 1). A linear wave can be expressed by a common linear wave equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \tag{1}$$

where t is the time, ∂ is a mathematical partial differential operator, and where c is the wave's velocity [2].

Solitons in the Korteweg–de Vries (KdV) equation (Equation 2) however, can retain their pulse form as they propagate because the nonlinearity cancels out that dispersion [1]. The KdV nonlinear equation solves for a faster (taller) soliton overtaking a slower (shorter) soliton as they collide when both moving in the same direction, for example, from left to right. One KdV equation can be described by:

$$\frac{\partial \mathrm{U}}{\partial \tau} + a \cdot U \frac{\partial \mathrm{U}}{\partial \zeta} + b \frac{\partial^3 \mathrm{U}}{\partial \zeta^3} = 0, \tag{2}$$

where ζ and τ are self-determining variables, a and b are constants, and ∂ is the mathematical operator partial differential. This form is used to describe long water waves in constant depth as well as one dimensional acoustic signals [3].

This property of retaining pulse form makes nonlinear waves particularly useful for transferring energy with less loss due to the lack of dispersion. Practical applications consist of medical uses such as non-invasive tools for eradicating specific cells by focusing these waves together in nonlinear beam-forming arrays [4]. The military may find uses during non-invasive strikes through air, water, or solids [5] or for mapping the ocean floor or detecting underwater objects [6]. Nonlinear waves are also useful for not just sound waves, but all types of waves; solitons can be found in gravitational [7], magnetic and electromagnetic waves as well [8]. Biological models have proposed that solitons are produced in proteins and DNA, and the brain sends signals using solitons [9]. Since they are not yet fully understood, more experimentation must be conducted to achieve a full understanding of nonlinear waves and produce equations to properly describe them.

The engineering goal of this experiment was to create a system capable of producing and recording both linear and nonlinear waves in order to achieve a greater understanding of nonlinear waves. There were two hypotheses of this experiment; the first hypothesis was that in a chain of spheres, the waves traveling throughout the chain would display nonlinear behaviors when struck with enough force to cause separation in the chain because the separation would not allow for the sound to reflect back through the system and disrupt the wave. The second hypothesis was that in a system consisting of a chain of spheres, the waves traveling throughout this chain would display nonlinear behaviors because the stress from the mass of the other spheres would cause deformation in the chain causing waves traveling in them to become nonlinear in nature.

Materials and Methods

A system for testing the wave behaviors was designed and engineered. To collect data, one 1.905 cm diameter solid steel sphere was cut in half and a piezoelectric sensor was permanently attached

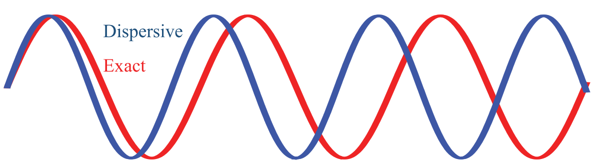


Figure 1. Numeric dispersion effect on a sinusoidal wave as it propagates. The sinusoidal wave and dispersive equivalent begin in phase, and over time the dispersive departs from the sinusoidal wave without affecting amplitude.

between the two half-spheres (Appendix A); this allowed the positive and negative leads from the sensor to protrude from the middle of the test sphere (Figure 2). Any dynamic force or strain applied to the test sphere was converted to an electrical charge relative to the change in pressure on the material from the waves traveling throughout the test sphere. The test sphere was put into a chain of four other solid steel spheres of equal diameter (1.905 cm) with masses of 28.246 g, which made a five-sphere chain in total for data sets 1 through 3, and with 14 other steel spheres for a total of 15 in data set 4. The spheres in the chain were identified and referred to in the order of 1 through 5 (bottom sphere = position 1 and top sphere = position 5) (Figure 3a). The chain was set vertically in a PVC pipe measuring 78.2 cm long with an inner diameter of 1.905 cm (Figure 3a). To ensure stability, the entire device was secured to a concrete wall.

The PVC housing had a 2.0 mm wide slot down the length to allow the positive and negative lead wires protruding from the test sphere to be connected to a Vernier LabQuest voltage probe (Figure 3b). For Data Sets 1 and 2, the probe was set to gather 1,000 samples per second for 3 seconds. For Data Sets 3 and 4, the probe was set to gather 10,000 samples per second for 1 second.

To create a wave through the chain in Data Sets 1 and 2, a spring-loaded striking system was engineered below the vertical chain of spheres and aligned to strike upwards (Figure 4). An incident wave pulse was generated by impact with the conformal striker on the position 1 sphere. An upwards striking motion was used to allow the position 5 sphere to separate from the rest of the chain, as indicated by the first hypothesis. The force applied by the striker was established using a Vernier LabQuest force sensor. The entire apparatus for the experimental trials is shown in Figure 5.

The material deformation (strain) for data collection was recorded on the voltage probe via the piezoelectric sensor within the test sphere. The data was then analyzed calculating the average voltage and standard deviation of each set of trials; the averages from all sets of data were statistically compared using a two-tailed t-test. The waveform was graphically generated to model the wave passing through the test sphere. The data was also analyzed using a new formula created for this research predicting nonlinearity:

$$\%Nonlinearity = 100 * \left(1 - \frac{A_S}{A_L}\right), \quad (3)$$

where A_L is the largest amplitude of an isolated wave, A_S is the lowest amplitude of the wave, and %Nonlinearity is the percent nonlinearity (Figure 6).

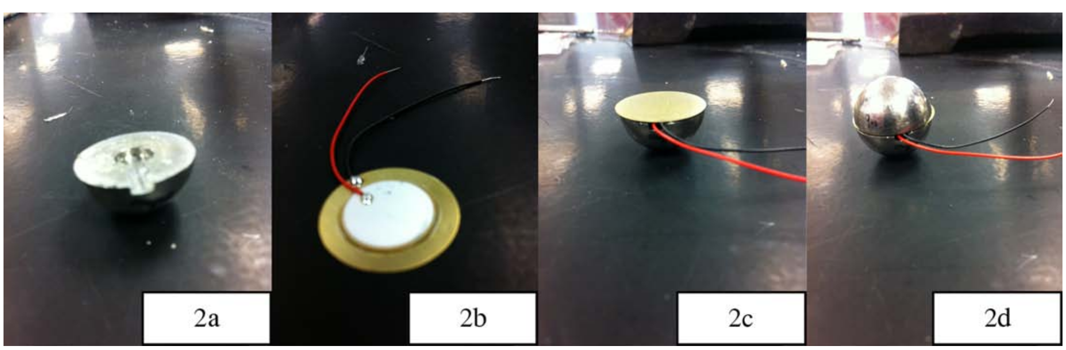


Figure 2. (2a) One side of the test sphere before assembly with a channel cut from the center out to allow for the wires from the (2b) piezoelectric sensor to (2c) protrude out for data collection and to ensure the force from trials did not become distorted. (2d) The fully assembled probe with protruding sensor leads for data collection.

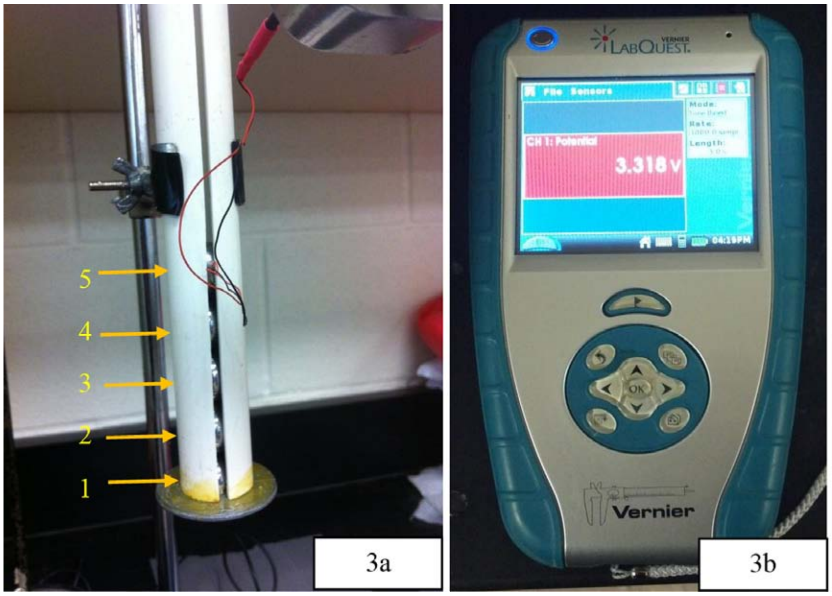


Figure 3. (3a) The test sphere placed at the top position in the 5 sphere chain; the chain was set vertically in a PVC pipe housing with the same inner diameter as the outer diameter of each sphere. The arrows with corresponding numbers reveal the different locations for the test sphere during trials. (3b) The Vernier LabQuest used for collecting data from the voltage probe.

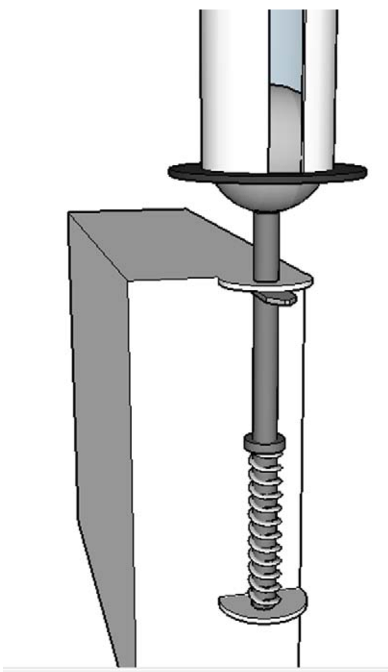


Figure 4. The impactor set to strike upwards touching the bottom sphere in the chain.

Figure 5. The system used to analyze sound waves consisting of a chain of varying numbers of steel spheres measuring 1.905 cm in diameter, a PVC pipe for holding the spheres in a line, a spring loaded striking system to ensure precise striking force, a test sphere adapted with a small piezoelectric sensor, and a Vernier LabQuest voltage probe for measuring the waveform via voltage amplitude from the test sphere.

The above systems and methodology were used during all experiments with these variations on the following data:

Data Set 1 (wave behaviors with varying force striking the chain)

The first trials were testing wave behaviors with different amounts of force striking the chain, as indicated by the first hypothesis. The test sphere was placed at the top of the chain (position 5) and the chain was struck with 5.0 N, 3.3 N, and 1.0 N of force. Ten trials were completed for each force application.

Data Set 2 (wave behaviors with F = 5.0 N, changing location of test sphere in the chain)

The second set of trials tested wave behaviors with a consistent force of 5.0 N, but changing the location of the test sphere in the chain, as indicated by the second hypothesis. Five trials were completed for each sphere position.

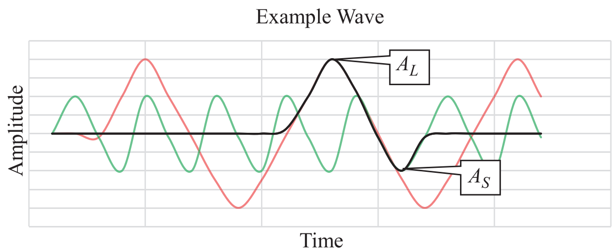


Figure 6. The A_L (absolute largest amplitude) and A_S (absolute smallest amplitude) shown on an example nonlinear wave. The black wave represents a nonlinear wave, the green is a hypothetical linear wave created using the lowest peak of that wave where A_S is the amplitude, and the red represents a hypothetical linear wave created by using the highest part of that peak where A_L is the amplitude of that wave. The nonlinearity is a ratio of the differences in amplitude of these red and green waves.

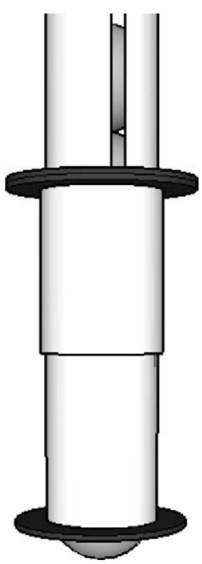


Figure 7. The compressed system for Data Set 3 connected to the system from Data Sets 1 and 2 to allow a falling sphere to strike the small compressed chain.

Data Set 3 (wave behaviors with compression applied to 5-sphere chain, changing location of test sphere in the chain)

The third set of trials tested wave behaviors in a compressed system similar to previous research [7]. The system was struck by releasing a sphere from 5.0 cm and letting it drop onto the chain. Five spheres including the test sphere were put into a compressed system so that there would not be any separation of the spheres during experimentation (Figure 7). There were trials without any added force and trials with 9.8 N of applied force similar to the previously mentioned research. 25 trials were completed for each sphere position and each force application, for a total of 250 trials.

Data Set 4 (wave behaviors with compression applied to 15-sphere chain, changing location of test sphere in the chain)

The methodology of Data Set 4 was completely similar to Data Set 3, but extending chain length from 5 spheres to 15 spheres. Samples were compiled into 10 trials for each sphere position for a total of 150 trials.

Results

Data Set 1 (wave behaviors with varying force striking the chain)

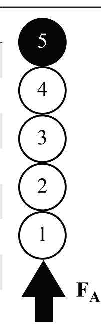
This data set consisted of multiple trials at 5.0 N of force on impact, 3.3 N of force, and 1.0 N of force. When analyzed using the measure for nonlinearity proposed in Eq. 3, on average the data had a nonlinearity of 97.35%±1.38%. The 3.3 N force trials had a high of 6.82 V with a low of -4.26 V. On average, the data had a nonlinearity of 80.70%±9.27%. The 1.0 N force trials had a high of 0.51 V with a low of -0.03 V. On average, the data had a nonlinearity of 64.83%±5.96% (Figure 8) (Table 1).

Data Set 2 (wave behaviors with F = 5.0 N, changing location of test sphere in the chain)

The data for set 2 consisted of trials with a consistent force of 5.0 N but different placements for the test sphere (Figure 3). The position 5 trials had a high of 7.96 V with a low of -12.11 V. When analyzed using the measure of nonlinearity proposed in

Table 1. Data Set 1 - The percent nonlinearity of each trial at 1.0 N, 3.3N, and 5.0 N of force with averages, calculated using Equation 3. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow. Sphere position 5 is the only position used in this data set.

Applied Force	1N	3.3N	5N
AVG Max (V)	3.54	4.77	4.84
AVG Min (V)	3.22	2.69	-0.38
AVG Mode (V)	3.32	3.32	3.32
AVG Difference (V)	0.33	2.08	5.22
AVG ABS High (V)	3.54	4.77	6.13
Nonlinearity (%)	64.83	80.70	97.35
Nonlinearity SD (%)	5.97	9.30	1.38
Trials (N) =	5	5	5



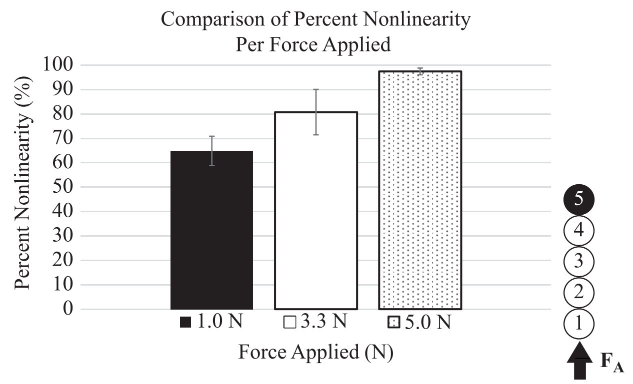


Figure 8. A graph showing the force applied on impact to the chain of spheres compared to the calculated nonlinearity from Data Set 1 with standard deviation. This supports the first hypothesis because as the force increased, the percent nonlinearity increased. This is thought to be from the separation of spheres on impact. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow. Sphere position 5 is the only position used in this data set.

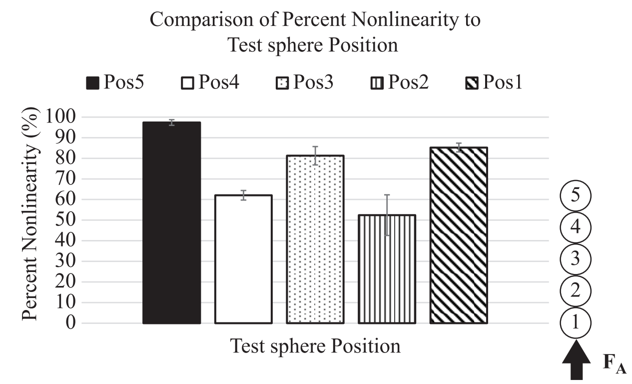
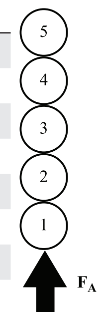


Figure 9. The percent nonlinearity compared to the test sphere position from Data Set 2 with standard deviation. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow.

Eq. 3, on average the data had a nonlinearity of 97.35%±1.38%. The position 4 trials had a high of 7.03 V with a low of -13.17 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 62.08%±2.32%. The position 3 trials had a high of 2.50 V with a low of -13.17 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 81.25%±4.43%. The position 2 trials had a high of 6.81 V with a low of -6.40 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 52.41%±9.82%. The position 1 trials had a high of 2.36 V with a low of -3.93 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 85.20%±2.12% (Figure 9) (Table 2).

Table 2. Data Set 2 - The percent nonlinearity of each trial at different test sphere locations through the chain with averages, calculated using Equation 3. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow.

Sphere Position	5	4	3	2	1
AVG Max (V)	4.84	3.81	3.68	5.99	5.37
AVG Min (V)	-0.38	2.84	2.07	-2.24	-6.57
AVG Mode (V)	3.32	3.31	3.31	3.31	3.31
AVG Difference (V)	5.22	0.97	1.60	8.22	11.94
AVG ABS High (V)	6.13	3.81	3.68	6.79	8.03
Nonlinearity (%)	97.35	62.08	81.26	52.42	85.21
Nonlinearity SD (%)	1.38	2.33	4.44	9.83	2.13
Trials (N) =	5	5	5	5	5



Data Set 3 (wave behaviors with compression applied to 5-sphere chain, changing location of test sphere in the chain)

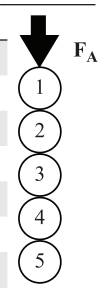
The data for set 3 consisted of trials with a consistent force of the falling sphere, but different placements for the test sphere, the first trials had no added force (control set) and the other had 9.8 N of applied force (experimental set) (Figure 7).

The first trials (without applied force) at position 5 had a high of 1.64 V with a low of -13.18 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 97.88%±1.12%. The position 4 trials had a high of 1.55 V with a low of -8.54 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 95.69%±0.97%. The position 3 trials had a high of 1.63 V with a low of -10.71 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 96.41%±7.01%. The position 2 trials had a high of 1.62 V with a low of -12.70 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 97.85%±1.95%. The position 1 trials had a high of 4.57 V with a low of -6.94 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 81.99%±10.47% (Table 3).

The second trials (with applied force) at position 5 had a high of 0.18 V with a low of -13.25 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 98.62%±0.54%. The position 4 trials had a high of 0.80 V with a low of -11.24 V. When analyzed using the nonlinear equation, on average the data

Table 3. Data Set 3 - The percent nonlinearity of each trial at different test sphere locations through the chain in the compressed system without added force, with averages calculated using Equation 3. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow.

Sphere Position	5	4	3	2	1
AVG Max (V)	0.18	0.34	0.19	0.14	3.66
AVG Min (V)	-9.71	-6.88	-8.78	-8.27	-1.38
AVG Difference (V)	9.89	7.22	8.97	8.41	5.04
AVG ABS High (V)	9.71	6.91	8.78	8.27	4.31
Nonlinearity (%)	97.88	95.69	96.41	97.85	81.99
Nonlinearity SD (%)	1.22	0.97	7.01	1.95	10.47
Trials (N) =	25	25	25	25	25



had a nonlinearity of 91.41%±5.14%. The position 3 trials had a high of 0.59 V with a low of -12.87 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 93.91%±3.12%. The position 2 trials had a high of 0.36 V with a low of -13.25 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 97.97%±0.83%.

The position 1 trials had a high of 0.52 V with a low of -9.64 V. When analyzed using the nonlinear equation, on average the data had a nonlinearity of 93.92%±2.77% (Figure 10) (Table 4).

Data Set 4 (wave behaviors with compression applied to 15-sphere chain, changing location of test sphere in the chain)

The data for set 4 consisted of trials with a consistent force of the falling sphere, but different placements for the test sphere, the trials had 9.8 N of applied force (Figure 11). The trials had a high nonlinearity of 98.82% and a low of 62.11%. Individual data averages and standard deviations can be found the data table (Table 5).

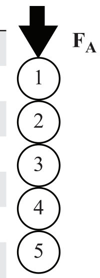
Discussion

This research was considered a success; a system capable of producing and recording both linear and nonlinear waves was created and was successful for testing. The first hypothesis was that in a chain of spheres, the waves traveling throughout the chain would display nonlinear behaviors when struck with enough force to cause separation in the chain because the separation would not allow for the sound to reflect back through the system and disrupt the wave. The hypothesis was accepted because the data showed a decrease in nonlinearity as the force applied decreased (Figure 8). The trials from Data Set 1 were analyzed using a two-tailed t-test, and there was a statistical difference between the 5.0 N and 3.3 N trials at the 90% confidence level ($t = \pm 3.77$; df = 8; $0.05 < p < 0.1$). There also was a statistical difference between the 3.3 N and 1.0 N trials at the 95% ($t = \pm 3.77$; df = 8; $0.01 < p < 0.05$). Between the 5.0 N and 1.0 N trials there was a statistical difference at the 99.9% confidence level ($t = \pm 4.77$; df = 8; $p < 0.001$). This supports the hypothesis since the increased difference between forces created higher nonlinearity. Future experimentation will test more severe differences in force with a hypothesis supporting the idea that there will be an even higher nonlinearity in higher force trials, and an even higher statistical difference.

The second hypothesis was that in a system consisting of a chain of spheres, the waves traveling throughout this chain would display nonlinear behaviors because the stress from the mass of the

Table 4. Data Set 3 - The percent nonlinearity of each trial at different test sphere locations through the chain in the compressed system with 9.8 N of added force, with averages calculated using Equation 3. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow.

Sphere Position	5	4	3	2	1
AVG Max (V)	0.12	0.63	0.48	0.21	0.39
AVG Min (V)	-9.18	-8.76	-8.97	-11.15	-7.23
AVG Difference (V)	9.30	9.38	9.45	11.36	7.61
AVG ABS High (V)	9.18	8.76	8.97	11.15	7.23
Nonlinearity (%)	98.62	91.41	93.91	97.97	93.92
Nonlinearity SD (%)	0.54	5.41	3.12	0.83	2.77
Trials (N) =	25	25	25	25	25



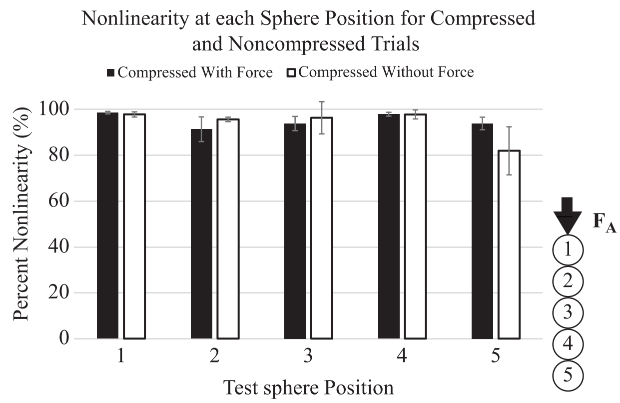


Figure 10. The trials from Data Set 3 showing nonlinearity compared to test sphere position for both no force compression and 9.8 N applied force compression trials with standard deviation. The figure to the right shows each sphere position in order and the direction of the applied striking Force (F_A) represented by the arrow.

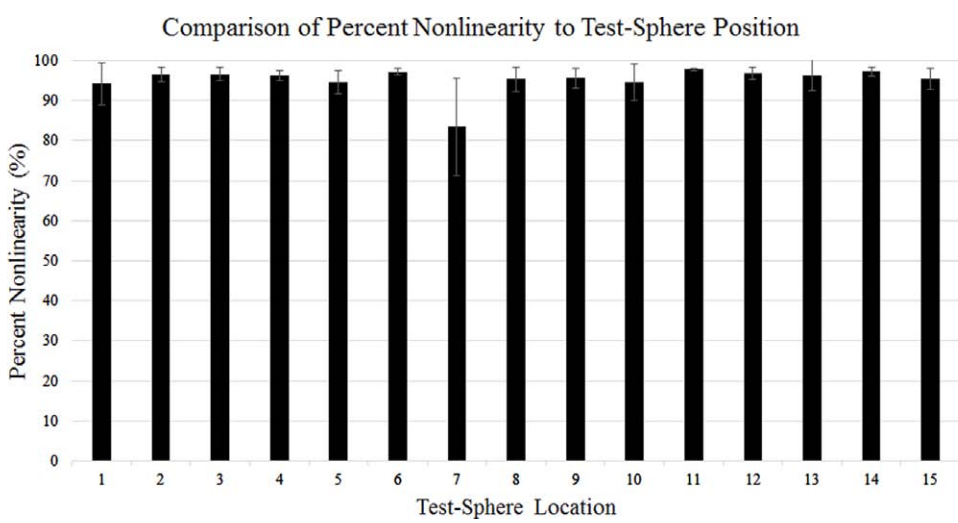


Figure 11. The trials from Data Set 4 showing nonlinearity compared to test sphere location with standard deviation. This shows a change in nonlinearity at each and the center of the array of spheres, possibly meaning there is some acoustic effect changing the waveform as it travels based on number of spheres.

Table 5. Data Set 4 - The average percent nonlinearity and standard deviation of that percentage shown per sphere position.

Sphere Position	**15**	**14**	**13**	**12**	**11**	**10**	**9**	**8**
Nonlinearity (%)	95.33	97.26	96.36	96.85	97.81	94.63	95.60	95.35
Nonlinearity SD (%)	2.64	1.14	3.38	1.45	0.34	4.63	2.56	2.98
Trials (N) =	10	10	10	10	10	10	10	10
Sphere Position	**7**	**6**	**5**	**4**	**3**	**2**	**1**	
Nonlinearity (%)	83.47	97.12	94.55	96.35	96.60	96.55	94.24	
Nonlinearity SD (%)	12.12	0.84	2.95	1.24	1.67	1.80	5.27	
Trials (N) =	10	10	10	10	10	10	10	

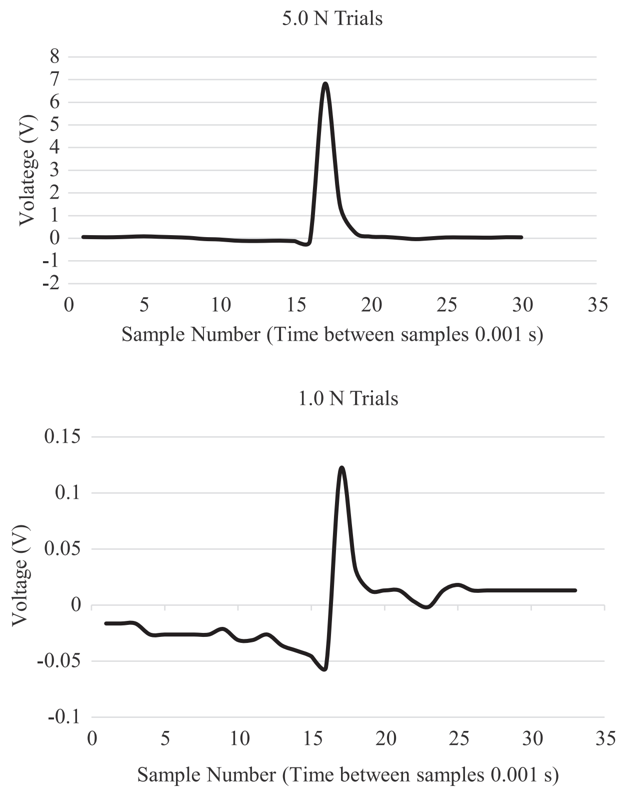


Figure 12. A graphical representation of a set of samples from the 5.0 N data set (Top) and the 1.0 N data set (Bottom) during Data Set 2. These graphs show that as the force applied is decreased, the nonlinearity decreases, as shown by the change in peak and trough of the wave over zero. The 5.0 N trial shows a voltage steady on zero and a peak that decreases back to zero, behaving like a soliton. The 1.0 N trial shows varied voltage in both the negative and positive values indicating a linear wave form. This supports the hypothesis in predicting that larger forces would produce higher nonlinearity percentages.

other spheres would cause deformation in the chain causing waves traveling in them to become nonlinear in nature. When compared using a two-tailed t-test, the different sphere positions were found to be statistically different at the 99.9% confidence level for both data sets 2 ($t = \pm 7.59$; df = 5; $p < 0.001$) and 3 ($t = \pm 5.24$; df = 25; $p < 0.001$), more so when getting further apart (Table 3, 4). (Figure 9, 11). This supports the hypothesis because higher forces applied to the lens statistically increased the nonlinearity.

Individual sample sets from Data Set 2 (5.0 N trials) initially revealed steady voltage set at zero with a peak which then decreased back to zero, which are behaviors like a solitons. Yet, the 1.0 N trials showed varied voltage in both the negative and positive values indicating a low nonlinearity. This supports the hypothesis in predicting that larger forces would produce higher nonlinearity percentages (Figure 12).

During Data Set 4, sphere position 7 was statistically different from all other sphere positions besides position 15 at least the 90% confidence level ($t = \pm 2.08$; df = 10; $0.05 < p < 0.1$). None of the other sphere positions when compared to each other showed a statistical difference; all statistical analysis was calculated with a two-tailed t-test.

In conclusion, this research was considered a success; both the first and second hypotheses were accepted and were significantly different. The testing produced highly nonlinear waves similar in shape to solitons, although more testing will be needed to compare the other aspects of solitons to this research. Future experimentation will be concentrated on application for solitons. Focused solitons could replace dangerous radiation therapy treatments or HIFU (High-Intensity Focused Ultrasound) due to its chance of increasing free radicals in the body. Focusing solitons could also create extremely high energy waves capable of many military applications and also sending sound waves long distances for discreet communication. More research will be conducted to determine the validity of these applications, but the data supports these methods being conceivable.

Acknowledgments

This research was conducted during my junior year at Odessa High School in the Advanced STEM Research Laboratory under the guidance of mentor Mr. Jeffery R. Wehr; I thank Mr. Wehr, the ASR Laboratory, and the Odessa High School for access to lab resources, technical supplies, and guidance preparing the scientific manuscript. I also acknowledge Dr. Andrew Ganse and Dr. Nick Boechler for technical support and advice during experimentation, as well as Dr. Chiara Daraio for donation of some experimental equipment. Finally, I would like to thank Julie Wehr for the support provided during the preparation of this research.

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Appendix A: Geometric and material properties of the steel spheres used in experimentation.

Diameter	D = 1.905 cm
Young's modulus	E = 200 GPa
Density	ρ = 7,800 kg/m^3
Poisson's ratio	v = 0.30
Mass	m = 28.246 g