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X Waves

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X Waves

- Introduction
  - Non-spreading waves – limited-diffraction beams
  - Invention of annular array Bessel transducer
  - Discovery of X waves
  - Study of X waves by others in physics
  - Further studies of X waves
- Applications of X waves
- Conclusion

Non-Spreading Waves – Limited-Diffraction Beams

- **Limited-Diffraction Beams**
  - Limited-diffraction beams are non-spreading and non-dispersive solutions to the isotropic-homogeneous wave equations. When produced with a finite aperture and energy in practice, these beams have a large depth of field for various applications. (Figure)
  - A limited-diffraction beam that is of a Bessel profile was called undistorted progressive waves by Stratton in his book in 1941.
  - In 1987, Durnin studied the Bessel beam again and produced the beam approximately with an optical experiment. Durnin called the Bessel beam non-diffracting or diffraction-free beam. Because all practical beams will eventually diffract, Durnin's terminology is controversial in scientific community.
  - To avoid the controversy, in 1992, we started to use the term "limited-diffraction beams."

Invention of Ultrasound Bessel Transducer

- To produce ultrasound Bessel beams, a transducer that is an annular array consisting of 10 annular rings was invented by us.
  - Jian-yu Lu and J. F. Greenleaf, "Ultrasonic nondiffracting transducer," *United States Patent*, no. 5081995, Issued: January 21, 1992.
  - Jian-yu Lu and J. F. Greenleaf, "Ultrasonic nondiffracting transducer for medical imaging," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 37, no. 5, pp. 438-447, September, 1990.
- This transducer was made of ceramic/polymer composite materials and had a 50 mm diameter, 2.5 MHz center frequency, and its ring widths were determined by the widths of the lobes of a J0 Bessel function. (Figures)

Discovery of X Waves

- Strictly speaking, Bessel beams are limited-diffraction at a single frequency (CW or monochromatic case).
- Many applications such as optics and ultrasound require a pulse of a short time duration (broadband).

Discovery of X Waves (Cont …)

- **X Waves**
  - X waves were discovered in 1991. These beams contain multiple frequencies, have a sharp peak intensity along the beam axis, exhibit characteristic branches in the shape of the letter "X", and propagate in space rigidly at a speed greater than the speed of sound in homogeneous media or speed of light in vacuum in theory. The X waves have the same phase and group velocity. X waves are one type of limited-diffraction beams. (Figure).

$$\Phi_{X_n}(\vec{r},t) = \Phi_{X_n}(r,\phi,z-c_1t)$$

$$= e^{in\phi}\int_0^\infty B(k)J_n(k\sin\zeta)e^{k[a_0 - i\cos\zeta(z-c_1t)]},$$

$$n = 1,2,3,\cdots,\ 0 \le \zeta < \pi/2,\ k = \omega/c$$

$$\frac{\omega}{k'} = \frac{\omega}{k\cos\zeta} = c_1 = c/\cos\zeta \ge c,$$

$$\frac{d\omega}{dk'} = \frac{d\omega}{dk\cos\zeta} = c_1 = c/\cos\zeta \ge c$$

Discovery of X Waves (Cont …)

- **Four Major Developments of X Waves**
  - Obtained systems of exact limited-diffraction solutions to the wave equation in 1991.
    - Jian-yu Lu and J. F. Greenleaf, "Nondiffracting X waves --- exact solutions to free-space scalar wave equation and their finite aperture realizations," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 39, no. 1, pp. 19-31, January, 1992.
    - Jian-yu Lu and J. F. Greenleaf, "Theory and acoustic experiments of nondiffracting X waves," in *1991 IEEE Ultrasonics Symposium Proceedings*, 91CH3079-1, vol. 2, pp. 1155-1159, 1991 (ISSN: 1051-0117).
  - Found a method to convert any solutions to the n-dimensional wave equation to an exact limited-diffraction solution to the n+1 dimensional wave equation in 1995. The method used is similar to the Lorentz transformation.
    - Jian-yu Lu, Hehong Zou and J. F. Greenleaf, "A new approach to obtain limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 42, no. 5, pp. 850-853, September, 1995.

Discovery of X Waves (Cont …)

- **Four Major Developments of X Waves (Continue …)**
  - Developed an X wave transformation by taking advantage of the orthogonal property of X waves in 2000. The transformation establishes both forward and inverse X wave transformations for any physically realizable waves.
    - Jian-yu Lu and Anjun Liu, "An X wave transform," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 47, no. 6, pp. 1472-1481, November, 2000.
  - New limited-diffraction solutions were obtained for the Klein-Gordon and Schrodinger equations in both free and confined spaces.
    - Jian-yu Lu, "Ultrasonic imaging with limited diffraction beams," in the book: *Localized Waves, Theory and Experiments* (submitted on October 30, 2005) (Invited by book editors appointed by the Editor-in-Chief of Progress in Electromagnetic Research to write a chapter. Also in *arXiv.org: physics/0603190*, pp. 1-25, March 23, 2006 ("physics/0603190" is a search keyword in Google).

Discovery of X Waves (Cont …)

- **Depth of Field of Limited-Diffraction Beams**
  - The concept of depth of field was introduced first in optics to describe the ability of a camera to get a sharp picture of objects located at different distances from the camera.
  - In both optical and ultrasound imaging, a large depth of field is important to bring objects in various distances into focus.

X Wave Studies by Others in Physics

- **X Waves in Nonlinear Optics**
  - Recently, there have been a lot of interests of applications of X waves in nonlinear optics. See the Search and Discovery column in:
    - Charles Day, "Intense X−Shaped Pulses of Light Propagate Without Spreading in Water and Other Dispersive Media", *Physics Today*, v.57, n.10, pp.25-26, October, 2004.
- **Some X wave related researches that appeared in the Journals of American Physical Society (APS):**
  - Physical Review Letters
    - Phys. Rev. Lett. 96, 193901 (2006)
    - Phys. Rev. Lett. 93, 153902 (2004)
    - Phys. Rev. Lett. 92, 253901 (2004)
    - Phys. Rev. Lett. 92, 120404 (2004)
    - Phys. Rev. Lett. 91, 093904 (2003)
    - Phys. Rev. Lett. 90, 170406 (2003)

X Wave Studies by Others in Physics (Cont…)

- Phys. Rev. Lett. 83, 1171–1174 (1999)
- Phys. Rev. Lett. 79, 4135–4138 (1997)
- Phys. Rev. Lett. 70, 1401–1404 (1993)

- Physical Review B (Condensed Matter and Materials Physics)
  - Phys. Rev. B 70, 235123 (2004)
- Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)
  - Phys. Rev. E 71, 016603 (2005)
  - Phys. Rev. E 70, 035601(R) (2004)
  - Phys. Rev. E 69, 066606 (2004)
  - Phys. Rev. E 69, 056611 (2004)
  - Phys. Rev. E 69, 036612 (2004)
  - Phys. Rev. E 69, 036608 (2004)
  - Phys. Rev. E 69, 027602 (2004)
  - Phys. Rev. E 69, 026607 (2004)
  - Phys. Rev. E 69, 016606 (2004)
  - Phys. Rev. E 68, 066612 (2003)
  - Phys. Rev. E 68, 026612 (2003)
  - Phys. Rev. E 68, 026610 (2003)

X Wave Studies by Others in Physics (Cont…)

- Phys. Rev. E 68, 016613 (2003)
- Phys. Rev. E 68, 016606 (2003)
- Phys. Rev. E 67, 066604 (2003)
- Phys. Rev. E 67, 056609 (2003)
- Phys. Rev. E 67, 036620 (2003)
- Phys. Rev. E 66, 046617 (2002)
- Phys. Rev. E 66, 056611 (2002)
- Phys. Rev. E 66, 046626 (2002)
- Phys. Rev. E 65, 046622 (2002)
- Phys. Rev. E 65, 026606 (2002)
- Phys. Rev. E 64, 066603 (2001)
- Phys. Rev. E 63, 046604 (2001)
- Phys. Rev. E 63, 026601 (2001)
- Phys. Rev. E 62, 5729–5737 (2000)
- Phys. Rev. E 62, 4261–4275 (2000)
- Phys. Rev. E 61, 2038–2041 (2000)
- Phys. Rev. E 58, 6742–6745 (1998)
- Phys. Rev. E 54, 4347–4352 (1996)
- Phys. Rev. E 48, 1410–1417 (1993)

Further Studies of Limited-Diffraction Beams

- **Limited-Diffraction Beams Have Been Further Studied by Us. For Example:**
  - Reduce sidelobe effects of limited-diffraction beams in imaging (Figure)
    - Jian-yu Lu, "Bowtie limited diffraction beams for low-sidelobe and large depth of field imaging," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 42, no. 6, pp. 1050-1063, November, 1995.
    - Jian-yu Lu, "Producing bowtie limited diffraction beams with synthetic array experiment," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 43, no. 5, pp. 893-900, September, 1996.
  - Harmonic productions of limited-diffraction beams in tissues:
    - Desheng Ding and Jian-yu Lu, "Second Harmonic Generation of the nth-order Bessel beams," *Physical Review E*, vol. 61, no. 2, pp. 2038-2041, February, 2000
    - Desheng Ding and Jian-yu Lu, "Higher-order harmonics of limited diffraction Bessel beams," *Journal of Acoustical Society of America*, vol. 107, no. 3, pp. 1212-1214, March, 2000.

Further Studies of Limited-Diffraction Beams (Cont…)

- **Limited-diffraction Beams Have Been Further Studied by Us. For Example (Continue …):**
  - Limited-diffraction array beams
    - Jian-yu Lu, "Limited diffraction array beams," *International Journal of Imaging System and Technology*, vol. 8, no. 1, pp. 126-136, January, 1997 (ISSN: 0899-9457).
  - Designing limited-diffraction beams of desired properties
    - Jian-yu Lu, "Designing limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 44, no. 1, pp. 181-193, January, 1997.
  - 2D arrays for producing limited-diffraction beams
    - Jian-yu Lu and J. F. Greenleaf, "A study of two-dimensional array transducers for limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 41, no. 5, pp. 724-739, September, 1994.

Applications of Limited-Diffraction Beams

- Efficient computation of wave fields with limited-diffraction beams
  - Jian-yu Lu and Jiqi Cheng, "Field computation for two-dimensional array transducers with limited diffraction array beams," *Ultrasonic Imaging*, vol. 27, no. 4, pp. 237-255, October, 2005.
- Pulse-echo imaging with a large depth of field (Figures)
  - Jian-yu Lu, M. Fatemi, and J. F. Greenleaf, "Pulse-echo imaging with X wave," in *Acoustical Imaging*, vol. 22, Piero Tortoli, Editor, pp. 191-196, 1996 (ISBN: 0-306-45364-9).
- Tissue characterization
  - Jian-yu Lu and J. F. Greenleaf, "Evaluation of a nondiffracting transducer for tissue characterization," in *1990 IEEE Ultrasonics Symposium Proceedings*, 90CH2938-9, vol. 2, pp. 795-798, 1990 (ISSN: 1051-0117).

Applications of Limited-Diffraction Beams (Cont…)

- Measurement of transverse velocity of blood flow (Figures)
  - Jian-yu Lu, "Improving accuracy of transverse velocity measurement with a new limited diffraction beam," in *1996 IEEE Ultrasonics Symposium Proceedings*, 96CH35993, vol. 2, pp. 1255-1260, 1996 (ISSN: 1051-0117).
  - Jian-yu Lu, Xiao-Liang Xu, Hehong Zou, and J. F. Greenleaf, "Application of Bessel beam for Doppler velocity estimation," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 42, no. 4, pp. 649-662, July, 1995.
- Nondestructive evaluation of materials (NDE) (Figures)
  - Jian-yu Lu and J. F. Greenleaf, "Producing deep depth of field and depth-independent resolution in NDE with limited diffraction beams," *Ultrasonic Imaging*, vol. 15, no. 2, pp. 134-149, April, 1993.
- Laser optical communications (Figures)
  - Jian-yu Lu and Shiping He, "Optical X wave communications," *Optics Communications*, vol. 161, pp. 187-192, March 15, 1999.

Applications of Limited-Diffraction Beams (Cont…)

- Optical coherent tomography (OCT) or optical diffraction tomography (ODT) (Figures)
  - Jian-yu Lu, Jiqi Cheng, and Brent Cameron, "Low sidelobe limited diffraction optical coherence tomography," in *Coherence Domain Optical Methods in Biomedical Science and Clinical Applications VI*, Valery V. Tuchin, Joseph A. Izatt, James G. Fujimoto, Editors, Proceedings of SPIE, vol. 4619, pp. 300-311, 2002 (ISBN: 0-8194-4358-1).
- Development of high frame rate 2D and 3D imaging methods for fast moving objects such as the heart and blood flow inside the heart (Figures)
  - Jian-yu Lu, "2D and 3D high frame rate imaging with limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 44, no. 4, pp. 839-856, July, 1997.
  - Jian-yu Lu, "Experimental study of high frame rate imaging with limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 45, no. 1, pp. 84-97, January, 1998.

Applications of Limited-Diffraction Beams (Cont…)

- High frame rate 2D and 3D imaging with a cylindrical wave transmission (Figures)
  - Hu Peng and Jian-yu Lu, "High frame rate 2D and 3D imaging with a curved or cylindrical array," in *2002 IEEE Ultrasonics Symposium Proceedings*, 02CH37388, vol. 2, pp. 1725-1728, 2002 (ISSN: 1051-0117).
- Construction of a high frame rate imaging system (Figures)
  - Jian-yu Lu, Jiqi Cheng, and Jing Wang, "High frame rate imaging system for limited diffraction array beam imaging with square-wave aperture weightings," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 53, no. 10, pp. 1796-1812, October, 2006.
  - Jian-yu Lu and John L. Waugaman, "Development of a linear power amplifier for high frame rate imaging system," in *2004 IEEE Ultrasonics Symposium Proceedings*, 04CH37553C, vol. 2, pp. 1413-1416, 2004 (ISSN: 1051-0117).
  - Jian-yu Lu, "A multimedia example," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol.50, no. 9, pp. 1078, September, 2003.

Applications of Limited-Diffraction Beams (Cont…)

- Extended HFR imaging and its motion, phase aberration, and noise effects (Figures)
  - Jian-yu Lu, Jiqi Cheng, and Jing Wang, "High frame rate imaging system for limited diffraction array beam imaging with square-wave aperture weightings," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 53, no. 10, pp. 1796-1812, October, 2006.
  - Jiqi Cheng and Jian-yu Lu, "Extended high frame rate imaging method with limited diffraction beams," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 53, no. 5, pp. 880-899, May, 2006.
  - Jing Wang and Jian-yu Lu, "Motion artifacts of extended high frame rate imaging," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* (Submitted).
  - Jing Wang and Jian-yu Lu, "Effects of phase aberration and noise on extended high frame rate imaging," *Ultrasound in Medicine and Biology* (Submitted).

Applications of Limited-Diffraction Beams (Cont…)

- Square-wave aperture weightings in transmission and reception (Figures)
  - Jian-yu Lu and Jing Wang, "Square-wave aperture weightings for reception beam forming in high frame rate imaging," in *2006 IEEE Ultrasonics Symposium Proceedings* (In Press).
- High frame rate blood flow velocity vector imaging (Figures)
  - Jian-yu Lu, Zhaohui Wang, and Sung-Jae Kwon, "Blood flow velocity vector imaging with high frame rate imaging methods," in *2006 IEEE Ultrasonics Symposium Proceedings* (In Press).

CONCLUSION

- **Limited-Diffraction Beams**
  - A unique type of solutions to the wave equation
  - Propagate to infinite distance without spreading in both propagation and transverse directions
  - In practice, they have a very large depth of field
  - May have theoretical significance in physics
  - Could have many potential applications

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