

FRACTALS ON THE LAND, AND BEYOND

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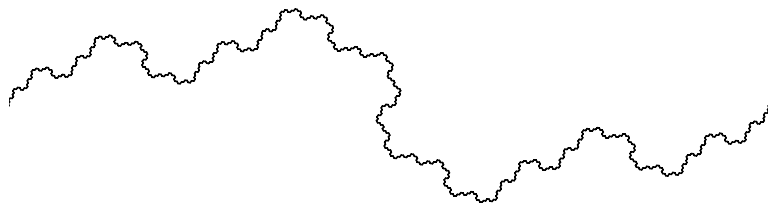
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- 1) **Boundaries with multi-scaled irregularity.** Many elements of landscape (drainage divides, ecosystem boundaries, etc.) are highly irregular curves with many overlaid scales of wandering. This richness in form, similar to that exhibited by biological membranes, city boundaries, and detailed ocean current traces, presents challenges for explanatory analysis and land management.



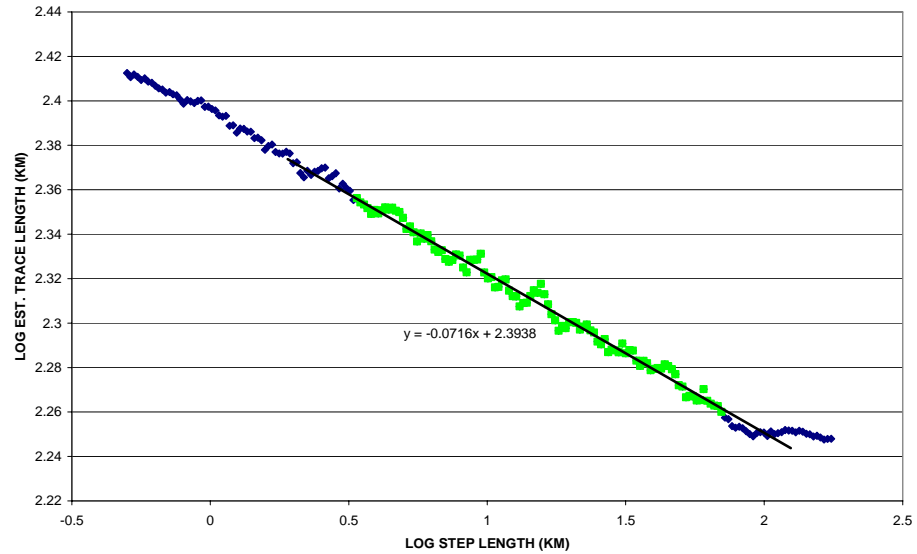
Trace of the North American Continental Divide.

- 2) **Paper exercise.** Rate the degrees of wandering in diverse curves.
- 3) **Artificial fractal models.** Koch curves and some other mathematical constructs display a strictly stratified form of self-similarity, with forms exactly repeated at different scales of view. Curve length increases without limit as construction proceeds, or with ever finer resolution of view, and the overall degree of curve wandering is quantified by the fractal dimension (D) value.



120° Koch curve, $D = 1.13$.

- 4) **Divider method.** In analysis of an irregular natural curve, the effects of varying resolution can be mimicked by walking a real or virtual map divider along the curve, varying divider step size to obtain different estimates of curve length. A logarithmic plot of results, a Richardson plot, shows degrees of curve wandering at specific scales by the plot slope values associated with particular step sizes. A plot or plot segment that is linear shows statistically self-similar character over the relevant range of scales, with D value obtained from the slope.
- 5) **Homogeneous fractal curves.** Some natural curves maintain a simple self-similar geometry expressed by consistent D value across all reliably examined scales of inspection. Examples cited here include a flank margin cave perimeter in Puerto Rico and portions of the boundary of most recent ice sheet advance in the Midwest. The fractal character of drainage basin boundaries creates difficulties for the calculation of perimeter-based basin form measures used in landscape analysis and flood prediction. It also calls attention to, and gives a basis for estimation of, error bounds in drainage divide delineation for watershed quality protection and management.

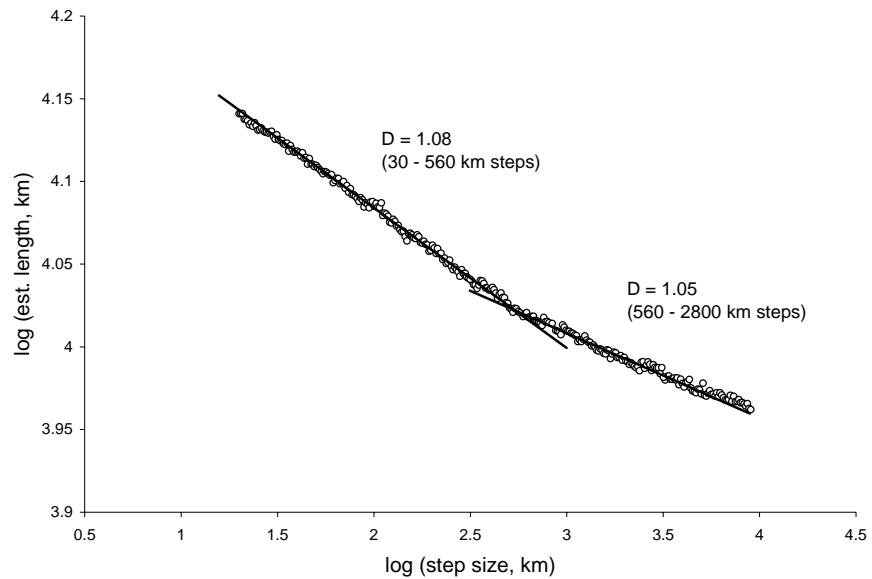


Richardson plot for the Wisconsin glacial limit in eastern Indiana and western Ohio (Adams, 2005).

- 6) **Heterogeneous fractal curves.** In many cases Richardson plots from natural curves show abrupt shifts in slope with changing resolution. These signify contrasts in curve geometry between two ranges of scale, and suggest scale thresholds in formative process. Heterogeneous fractal geometries express the effects of rock striation processes and less-understood, large-scale grooving processes in bedrock glacial erosion features. They clearly differentiate modern and paleohydraulic signatures in meandering river channels. Heterogeneous geometry also appears to mark the different scale ranges associated with tectonic and erosional sculpting of the North American Continental Divide.

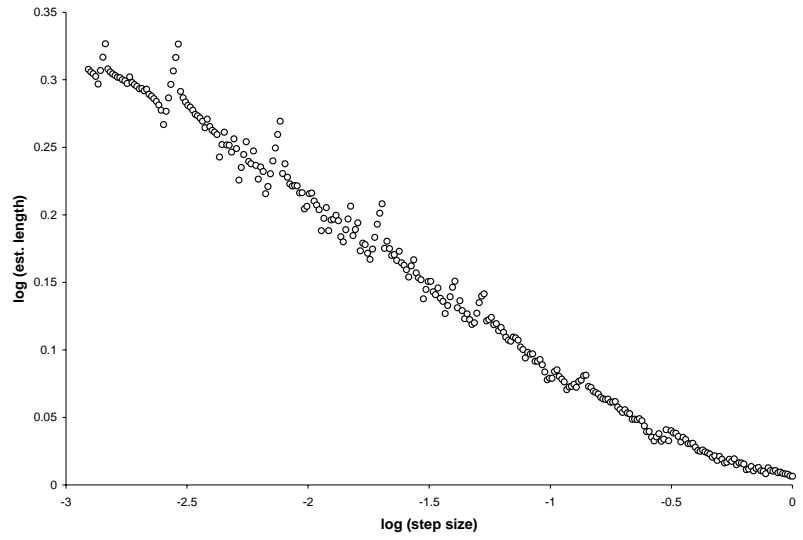


Using a contour gauge to record profiles of glacial striations and grooves in limestone, Kelleys Island, Ohio.

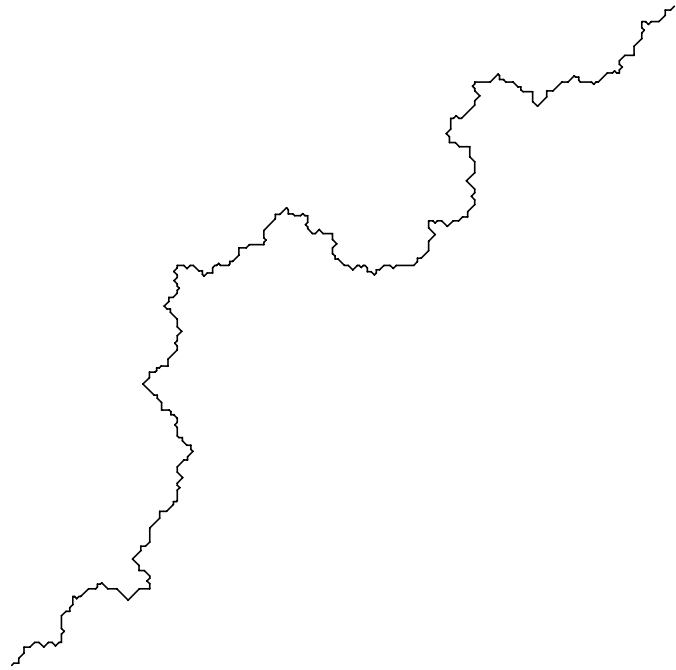


Richardson plot for North American Continental Divide trace.

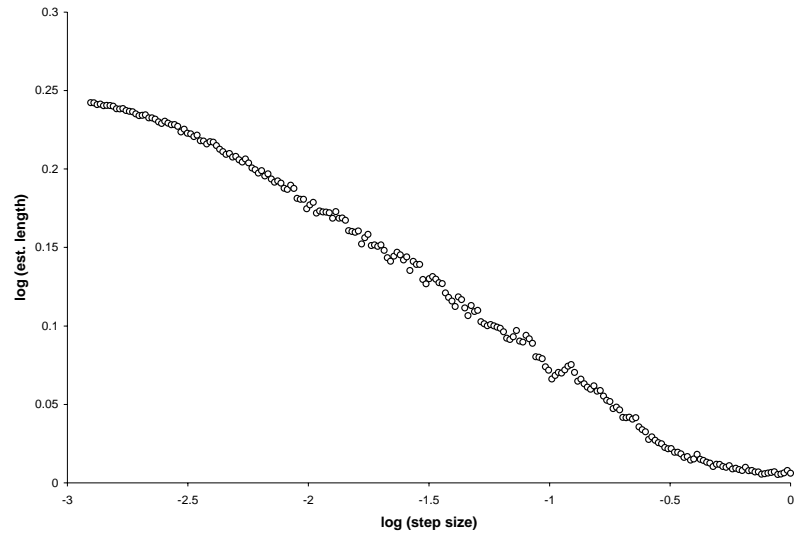
- 7) **Forum.** What are speculative and currently acknowledged implications of fractal boundary complexity in your area of study? How effectively do the typical, stock examples actually model the actual occurrences? What are the limitations of this type of modeling?
- 8) **Nature leads us toward improved artificial fractal models.** Abandonment of stratified constructions produces curves with more robust fractal character. The stratified nature of typical mathematical fractals produces anomalies under analysis. We were therefore led to create different mathematical models lacking such a stratified characteristic in effort to better model the naturally occurring fractals. Our distinctive process creates a curve with a designated fractal dimension which effectively distributes the necessary irregularity across all scales of inspection. In addition, our models expand the mathematical landscape of fractals from a collection of strictly regulated, scale-dependent examples to a richer, more useful collection.



The Problem: Richardson plot for 120° Koch curve, $D = 1.13$.



Nonstratified fractal curve, $D = 1.10$



Richardson plot for nonstratified fractal curve, $D = 1.10$